Appendix – Proof that real income growth equals money income growth minus the rate of inflation

Let

N = money income
P = index of average prices
R = real income = N/P

and

r = rate of change in R =$\dot{R}=\frac{dR}{dt}$
n = rate of change in N =$\dot{N}=\frac{dN}{dt}$
p = rate of change in average prices, also known as the inflation rate =$\dot{P}=\frac{dP}{dt}$

Now, let

g = the economy's real growth rate = r/R=$\frac{\dot{R}}{R}$
m = the economy's nominal (or money) growth rate = n/N=$\frac{\dot{N}}{N}$

Then, it is straightforward to show that:

$$g = m - p \text{ (or } g = m + d) \qquad \text{(with } d=-p) \qquad (1)$$

that is,

Real Growth Rate = Nominal Growth Rate *minus* Inflation Rate

OR

Real Growth Rate = Nominal Growth Rate *plus* Deflation Rate

Thus, in deflationary times (i.e. $d>0$), when prices fall faster than money incomes ($d>m$), real growth appears positive ($g>0$). For money incomes to be growing ($m>0$) during deflationary times, real growth (g) must exceed the rate of deflation (d).

Proof of Equation (1)

$R=N/P$. Thus $\dot{R}=\frac{P\dot{N}-N\dot{P}}{P^2}$. Substituting in $g=\frac{\dot{R}}{R}$, we get:

$$g=\frac{\dot{N}}{PR}-\frac{N}{P}\times\frac{\dot{P}}{P}\times\frac{1}{R}=\frac{\dot{N}}{P\frac{N}{P}}-\frac{N}{P}\times\frac{\dot{P}}{P}\times\frac{1}{\frac{N}{P}}=\frac{\dot{N}}{N}-\frac{\dot{P}}{P}=m-p \qquad \text{QED}$$